Functional Data Structures for Typed Racket

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Motivation

Typed Racket has very few data structures

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Lists

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Vectors

Motivation

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- Lists
- Vectors
- Hash Tables

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Typed Racket has very few data structures

- Lists
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Practical use of Typed Racket

Outline

- Motivation
- Typed Racket in a Nutshell
- Purely Functional Data Structures
- Benchmarks
- Typed Racket Evaluation
- Conclusion

Function definition in Racket

```
#lang racket

; Computes the length of a given list of elements
; length : list-of-elems -> natural
(define (length list)
  (if (null? list)
      0
      (add1 (length (cdr list)))))
```

Function definition in Typed Racket

```
#lang typed/racket

; Computes the length of a given list of integers
(: length : (Listof Integer) -> Natural)
(define (length list)
  (if (null? list)
      0
      (add1 (length (cdr list)))))
```

Function definition in Typed Racket

```
#lang typed/racket

; Computes the length of a given list of elements
(: length : (All (A) ((Listof A) -> Natural)))
(define (length list)
  (if (null? list)
      0
      (add1 (length (cdr list)))))
```

Data definition in Racket

```
#lang racket

; Data definition of tree of integers

; A Tree is one of
; - null
; - BTree

(define-struct BTree
  (left
   elem
   right))

; left and right are of type Tree
; elem is an Integer
```

Data definition in Typed Racket

```
#lang typed/racket

; Data definition of tree of integers

(define-type Tree (U Null BTree))


(define-struct: BTree
  ([left  : Tree]
   [elem  : Integer]
   [right : Tree]))
```

Data definition in Typed Racket

```
#lang typed/racket

; Polymorphic definition of Tree

(define-type (Tree A) (U Null (BTree A)))


(define-struct: (A) BTree
  ([left  : (Tree A)]
   [elem  : A]
   [right : (Tree A)]))
```

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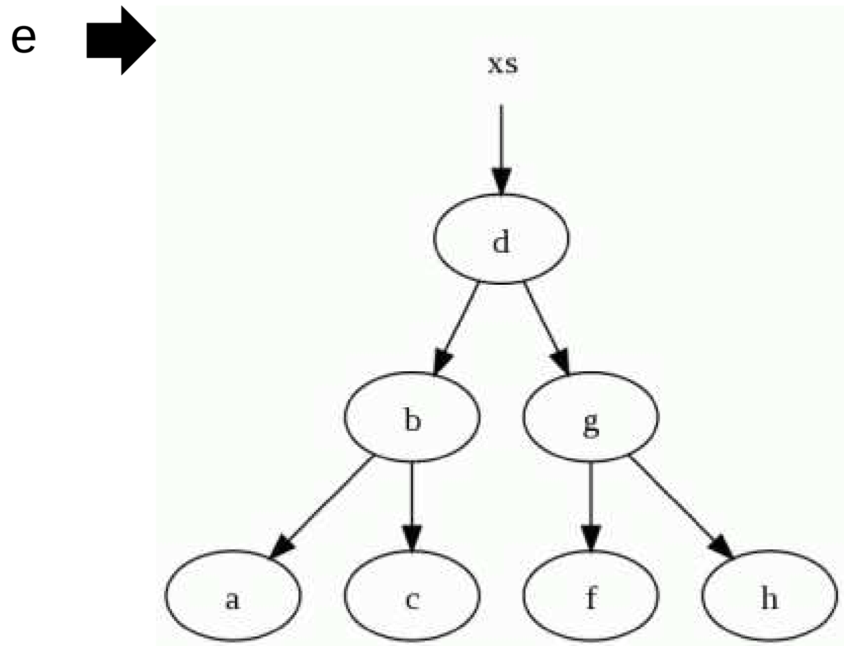
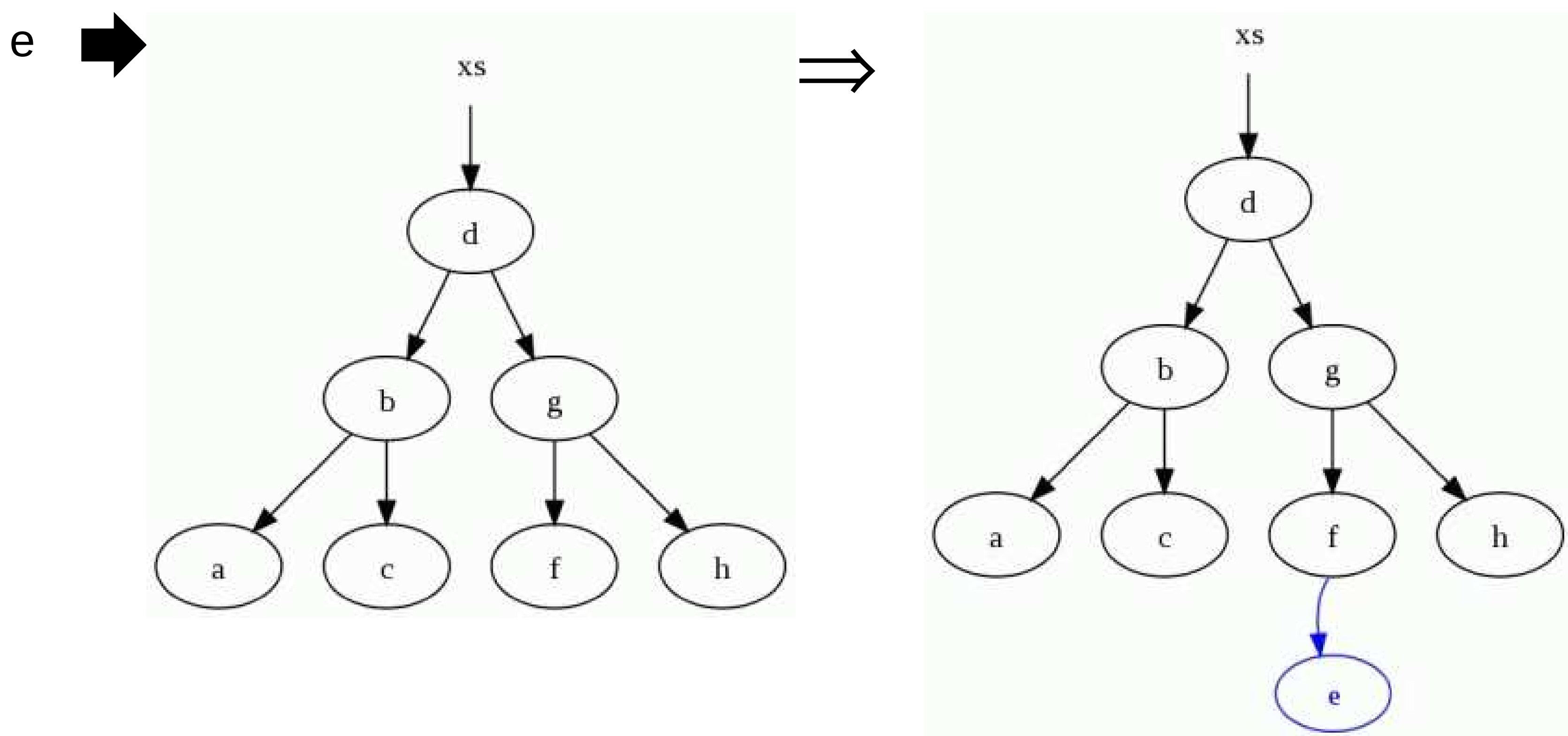
Destructive and Non-destructive update

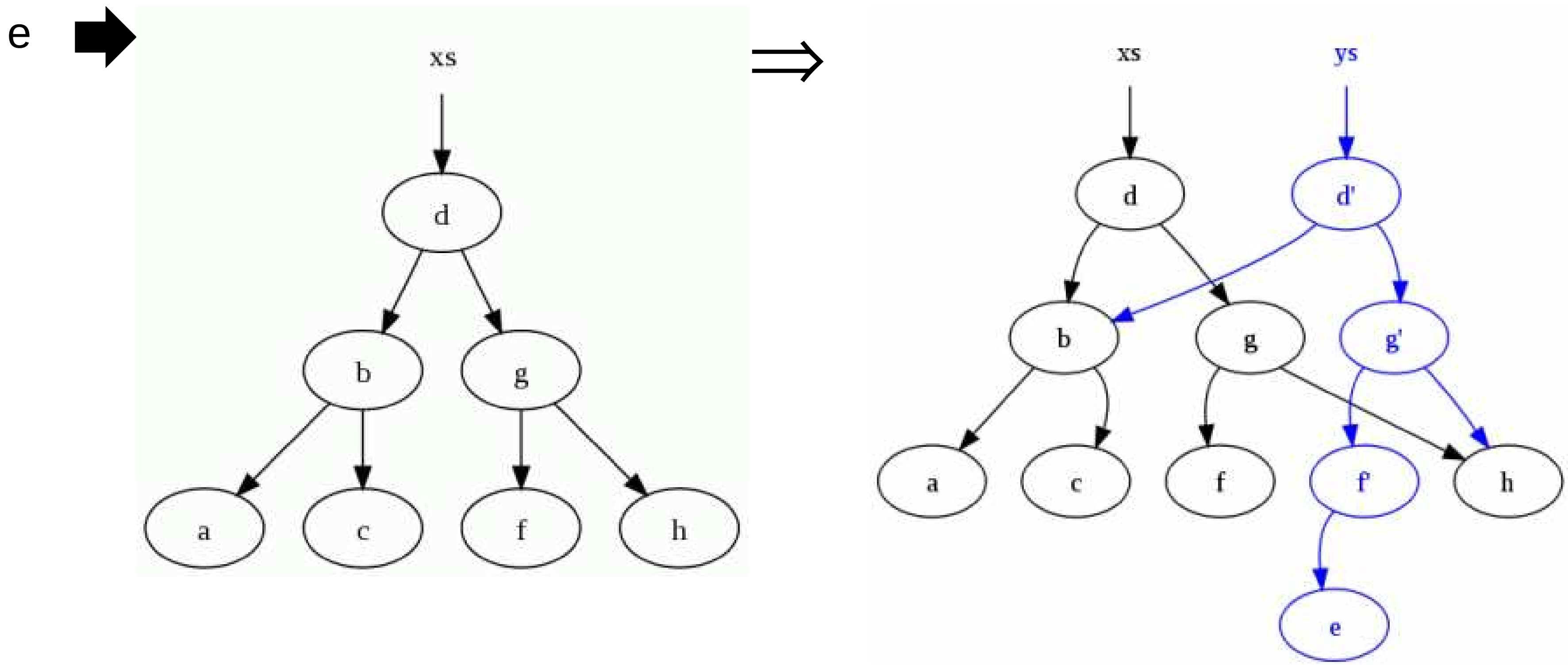


Destructive and Non-destructive update



Destructive update

Destructive and Non-destructive update



Non-destructive update

Functional Queue

```
(define-struct: (A) Queue
  ([front : (Listof A)]
   [rear  : (Listof A)]))
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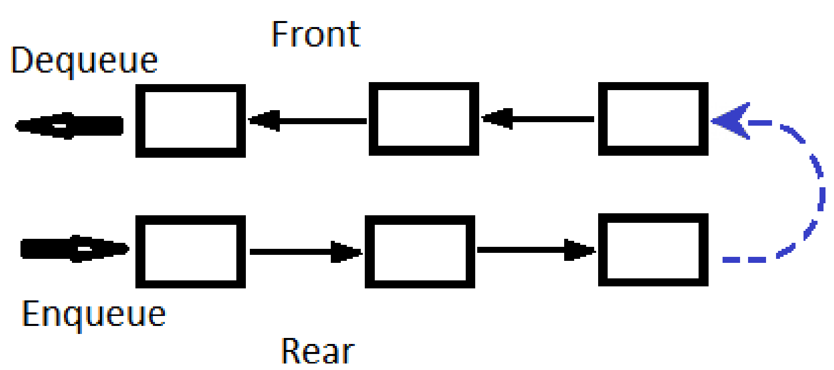
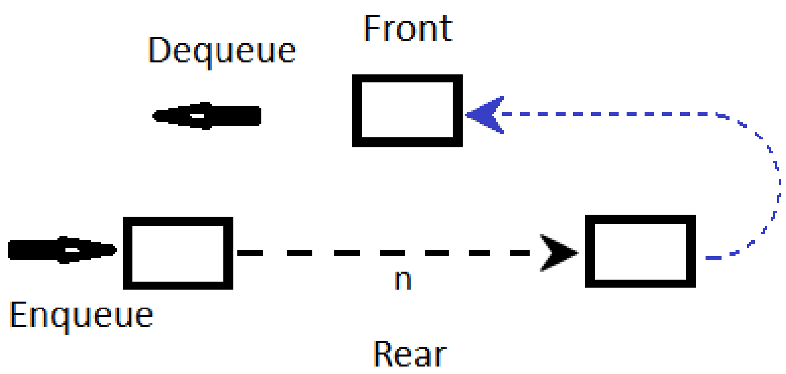


Functional Queue

```
(: dequeue : (All (A) ((Queue A) -> (Queue A))))
(define (dequeue que)
  (let ([front (cdr (Queue-front que))]
        [rear  (Queue-rear que)])
    (if (null? front)
        (Queue (reverse rear) null)
        (Queue front rear))))
```

Functional Queue

Queue **q**




```
(for ([id (in-range 100)])
  (dequeue q))
```

Banker's Queue [Okasaki 1998]

Lazy evaluation solves this problem

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Lazy evaluation solves this problem

```
(: val : (Promise Exact-Rational))
(define val (delay (/ 5 0)))
```

Banker's Queue [Okasaki 1998]

Lazy evaluation solves this problem

Streams

```
(define-type (Stream A)
  (Pair A (Promise (Stream A))))
```

Banker's Queue [Okasaki 1998]

Lazy evaluation solves this problem

```
(define-struct: (A) Queue
  ([front : (Stream A)]
   [lenf  : Integer]
   [rear  : (Stream A)]
   [lenr  : Integer]))
```

Invariant `lenf >= lenr`

Banker's Queue [Okasaki 1998]

Lazy evaluation solves this problem

```
(: check :
   (All (A) (Stream A) Integer (Stream A) Integer -> (Queue A)))

(define (check front lenf rear lenr)
  (if (>= lenf lenr)
      (make-Queue front lenf rear lenr)
      (make-Queue (stream-append front (stream-reverse rear))
                  (+ lenf lenr) null 0)))
```

Banker's Queue [Okasaki 1998]

Lazy evaluation solves this problem

```
(make-Queue (stream-append front (stream-reverse rear))
            (+ lenf lenr) null 0)
```

Banker's Queue [Okasaki 1998]

Lazy evaluation solves this problem

Amortized running time of O(1) for the operations **enqueue**, **dequeue** and **head**

Real-Time Queues [Hood & Melville 81]

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Eliminating amortization by Scheduling

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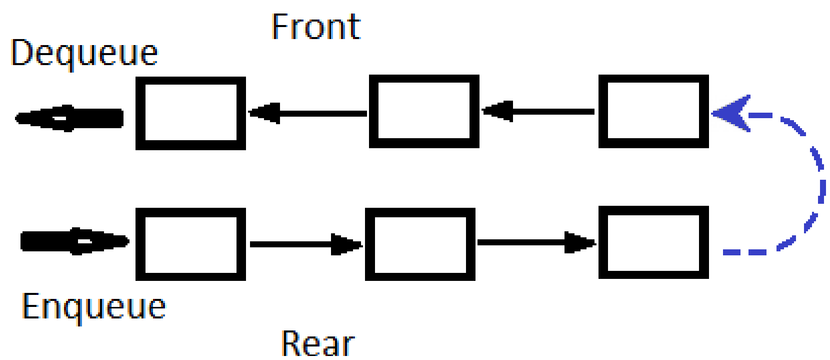

Banker's Queue - **`reverse`** is a forced completely

Real-Time Queues [Hood & Melville 81]

Eliminating amortization by Scheduling

```
(: rotate :
   (All (A) ((Stream A) (Listof A) (Stream A) -> (Stream A))))

(define (rotate front rear accum)
  (if (empty-stream? front)
      (stream-cons (car rear) accum)
      (stream-cons (stream-car front)
                   (rotate (stream-cdr front)
                           (cdr rear)
                           (stream-cons (car rear) accum)))))
```

Incremental reversing

Real-Time Queues [Hood & Melville 81]

Eliminating amortization by Scheduling

Worst-case running time of O(1) for the operations **enqueue**, **dequeue** and **head**

Binary Random Access Lists [Okasaki 1998]

```
Nat is one of
- 0
- (add1 Nat)

List is one of
- null
- (cons elem List)
```

Binary Random Access Lists [Okasaki 1998]

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Nat is one of
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```

`cons` corresponds to increment

`cdr` corresponds to decrement

`append` corresponds to addition

Binary Random Access Lists [Okasaki 1998]

```
(define-type (RAList A) (Listof (Digit A)))
```

Binary Random Access Lists [Okasaki 1998]

```
(define-type (RAList A) (Listof (Digit A)))
(define-type (Digit A) (U Zero (One A)))
```

Binary Random Access Lists [Okasaki 1998]

```
(define-struct: Zero ())
```

Binary Random Access Lists [Okasaki 1998]

```
(define-struct: Zero ())
(define-struct: (A) One ([fst  : (Tree A)]))
```

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```
(define-type (Tree A) (U (Leaf A) (Node A)))
```

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(define-type (Tree A) (U (Leaf A) (Node A)))
(define-struct: (A) Leaf ([fst : A]))
```

Binary Random Access Lists [Okasaki 1998]

```
(define-type (Tree A) (U (Leaf A) (Node A)))

(define-struct: (A) Leaf ([fst : A]))

(define-struct: (A) Node
  ([size  : Integer]
   [left  : (Tree A)]
   [right : (Tree A)]))
```

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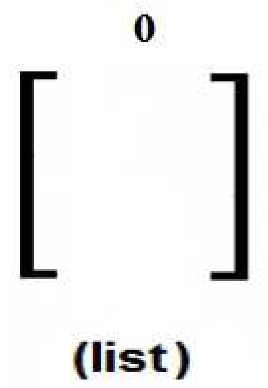

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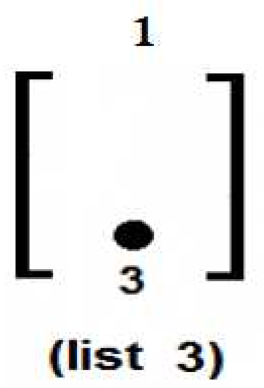

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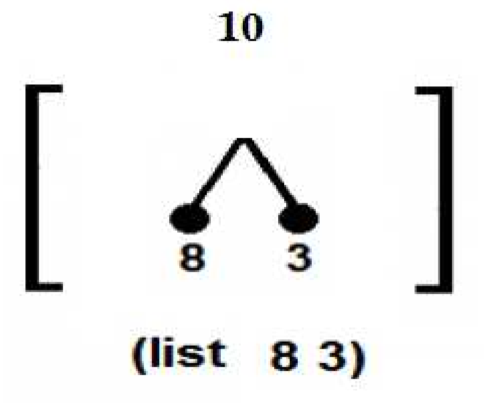

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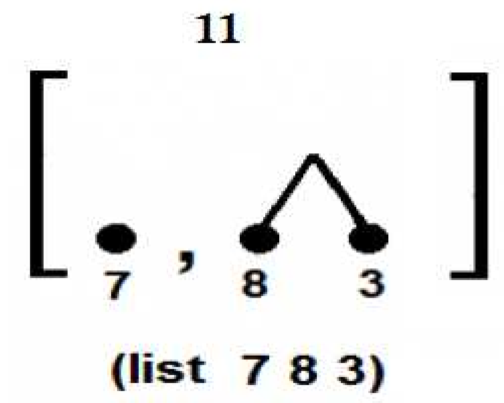

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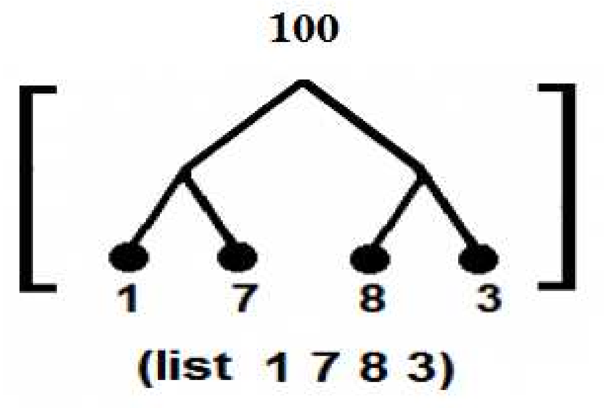

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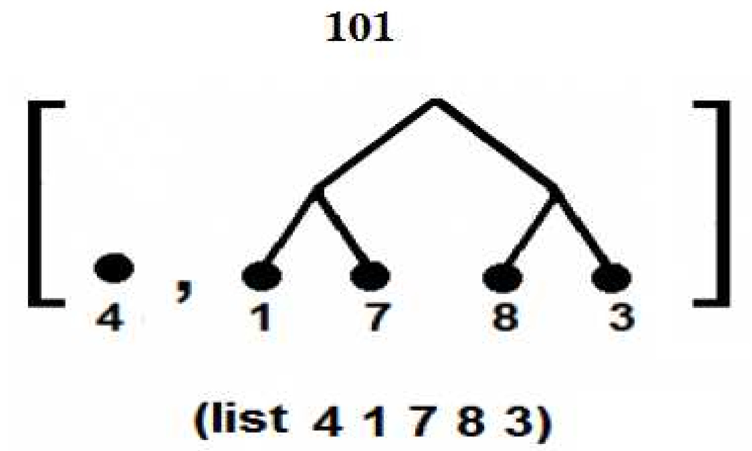

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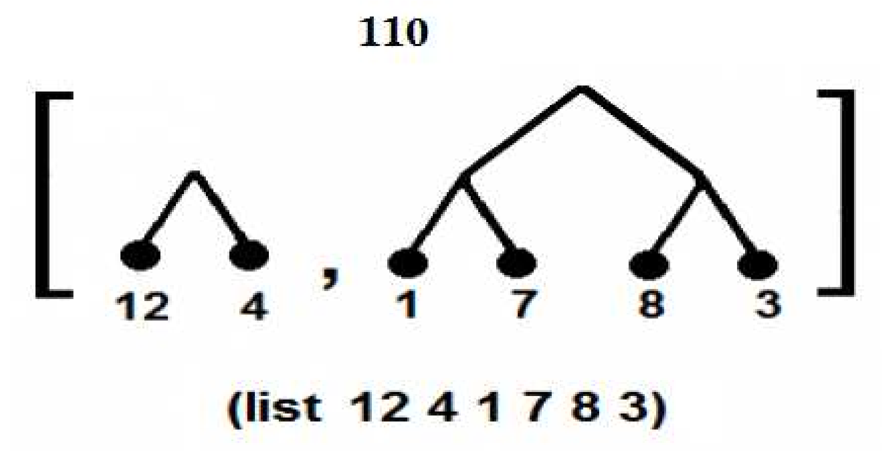

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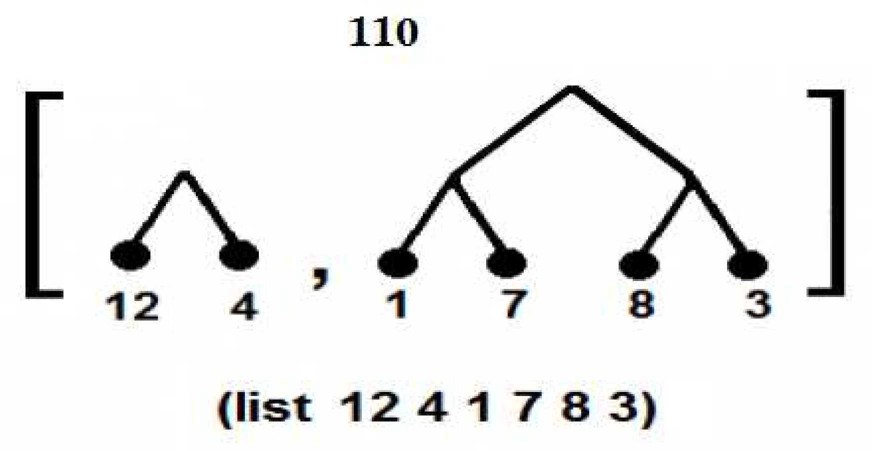


Worst-case running time of O(log n) for the operations

`cons`, **`car`**, **`cdr`**, **`lookup`** and **`update`**

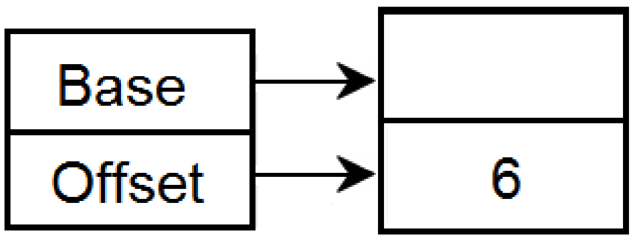
VLists [Bagwell 2002]

```
(define-struct: (A) Base
  ([previous : (U Null (Base A))]
   [elems    : (RAList A)]))


(define-struct: (A) VList
  ([offset : Natural]
   [base   : (Base A)]
   [size   : Natural]))
```

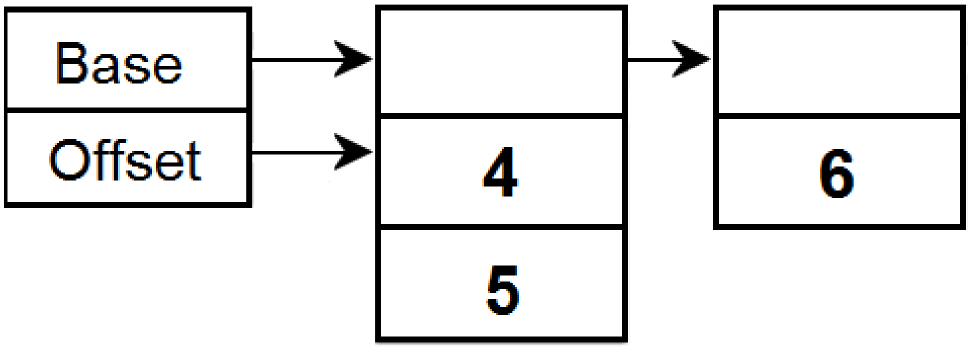
VLists [Bagwell 2002]

List with one element - 6



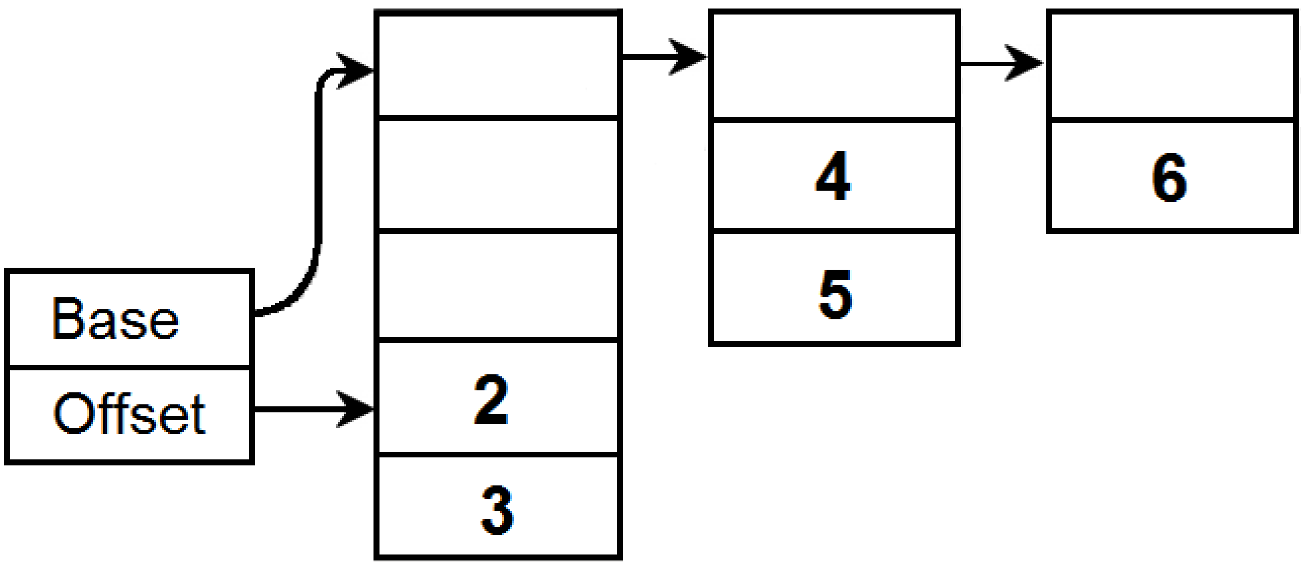
VLists [Bagwell 2002]

cons 5 and 4 to the previous list



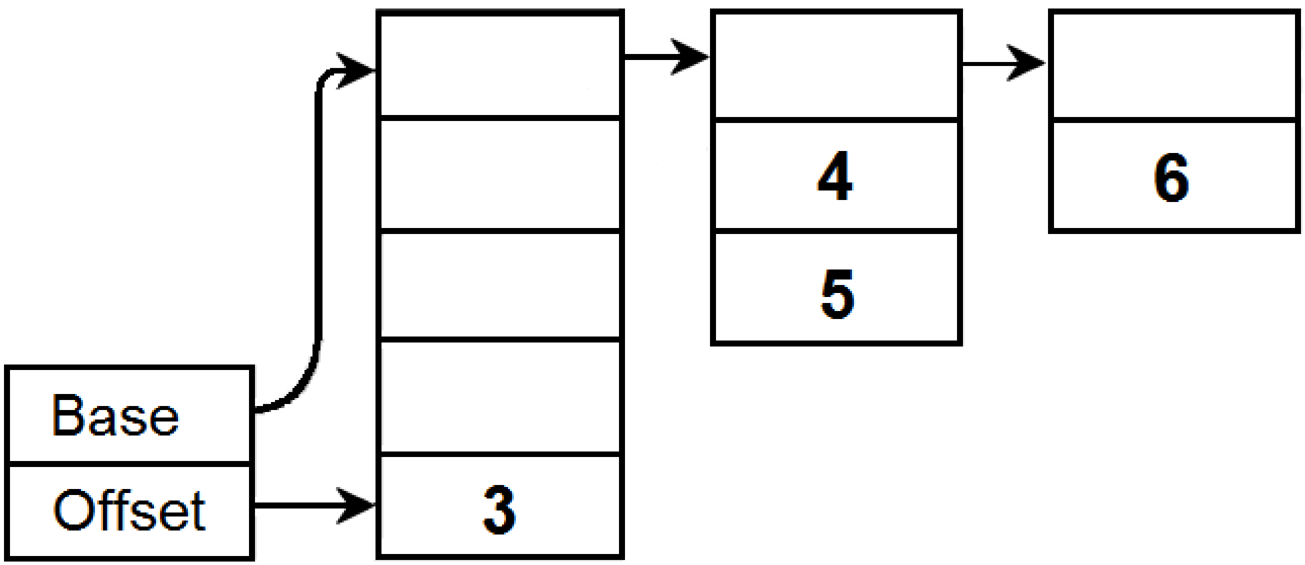
VLists [Bagwell 2002]

cons 3 and 2 to the previous list



VLists [Bagwell 2002]

`cdr` of the previous list



VLists [Bagwell 2002]

Random access takes O(1) average and O(log n) in worst-case.

Our library

Library has 30 data structures which include

- Variants of Queues
- Variants of Deques
- Variants of Heaps
- Variants of Lists
- Red-Black Trees
- Tries
- Sets
- Hash Lists

Our library

Library has 30 data structures

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Data structures have several utility functions

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Data structures have several utility functions

Our implementations follows the original work

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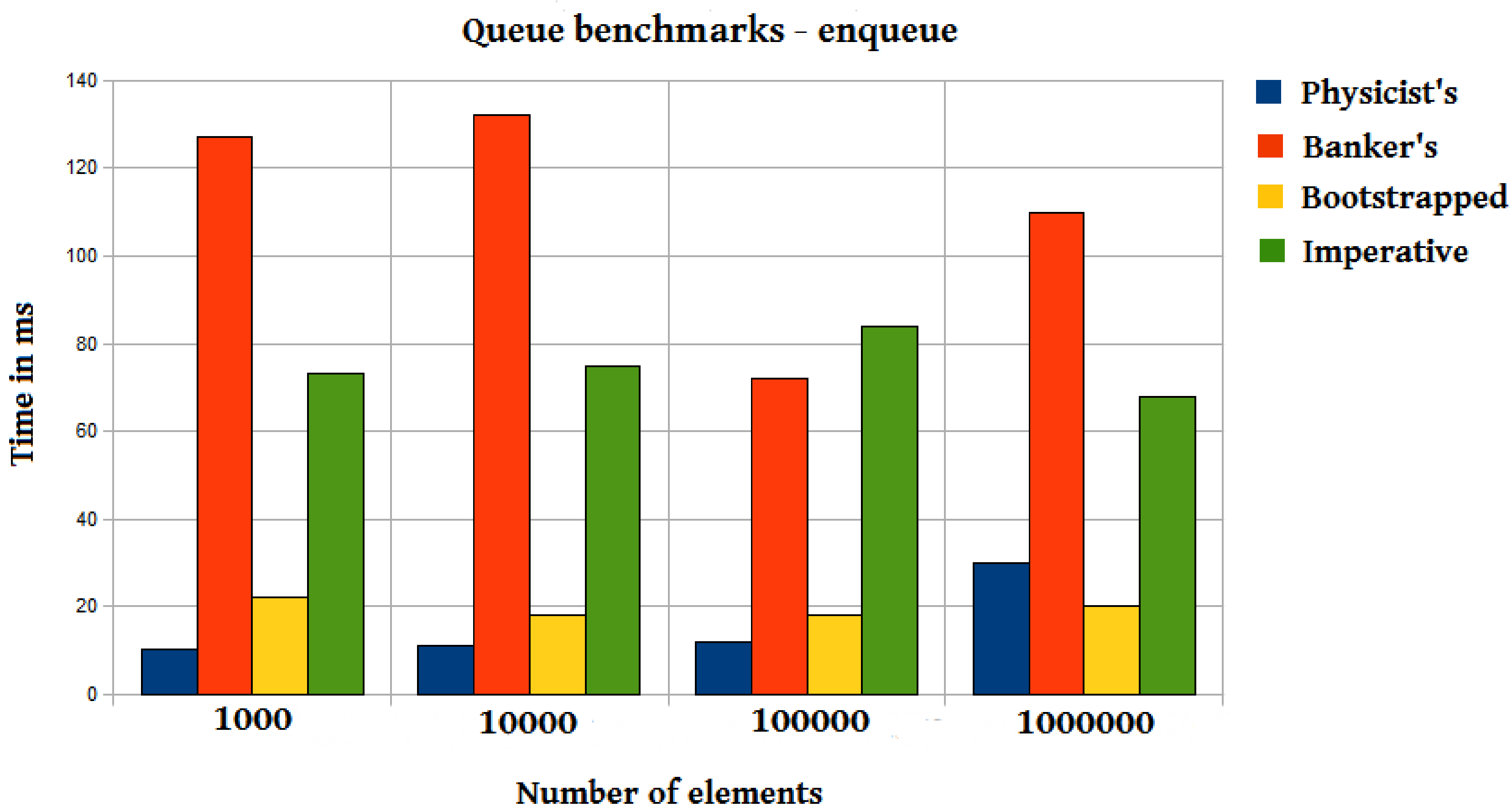
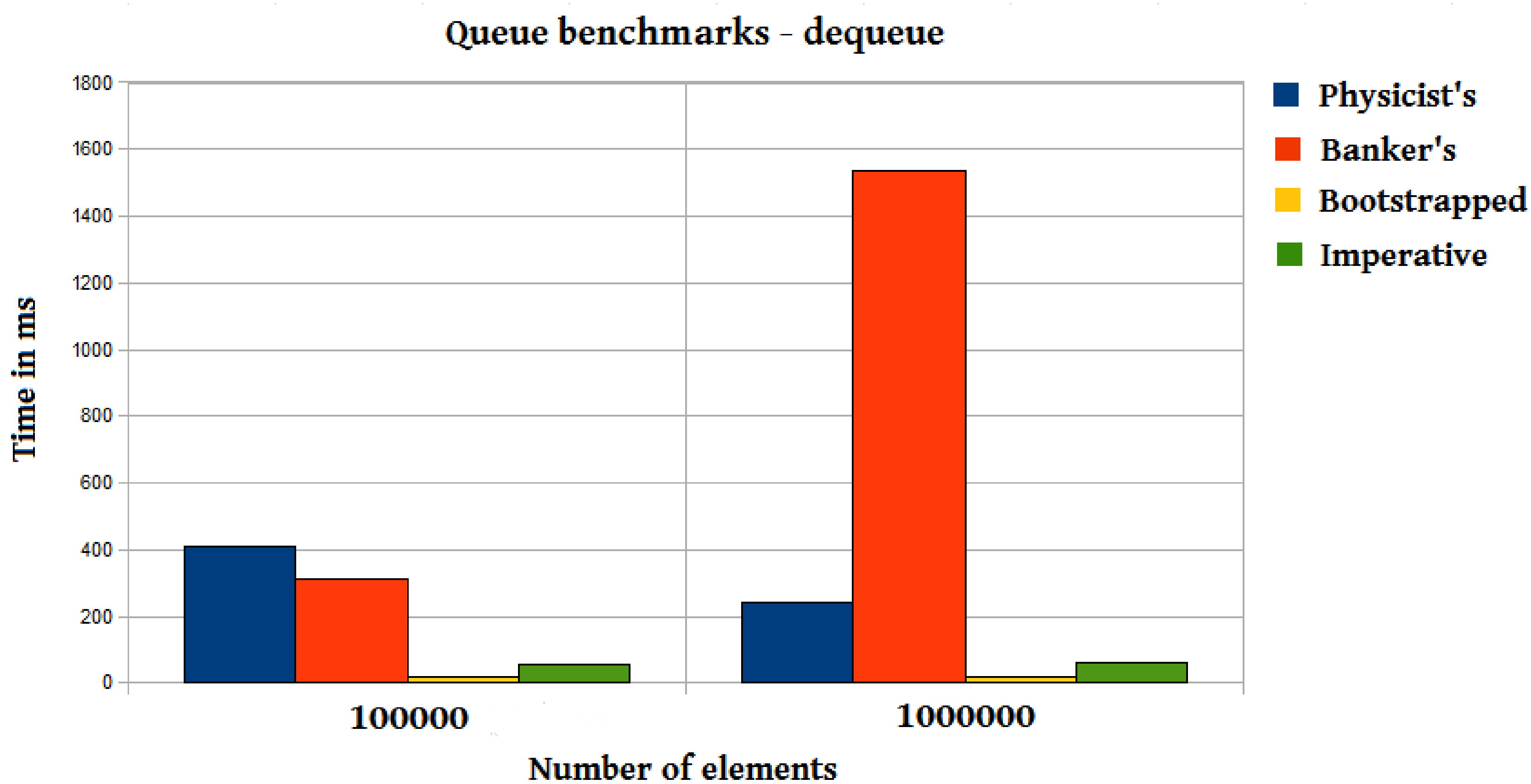
Benchmarks



** Benchmarking done with 2.1 GHz Intel Core 2 Duo (Linux) machine using Racket version 5.0.0.9

```
(foldl enqueue que list-of-100000-elems)
```

Benchmarks



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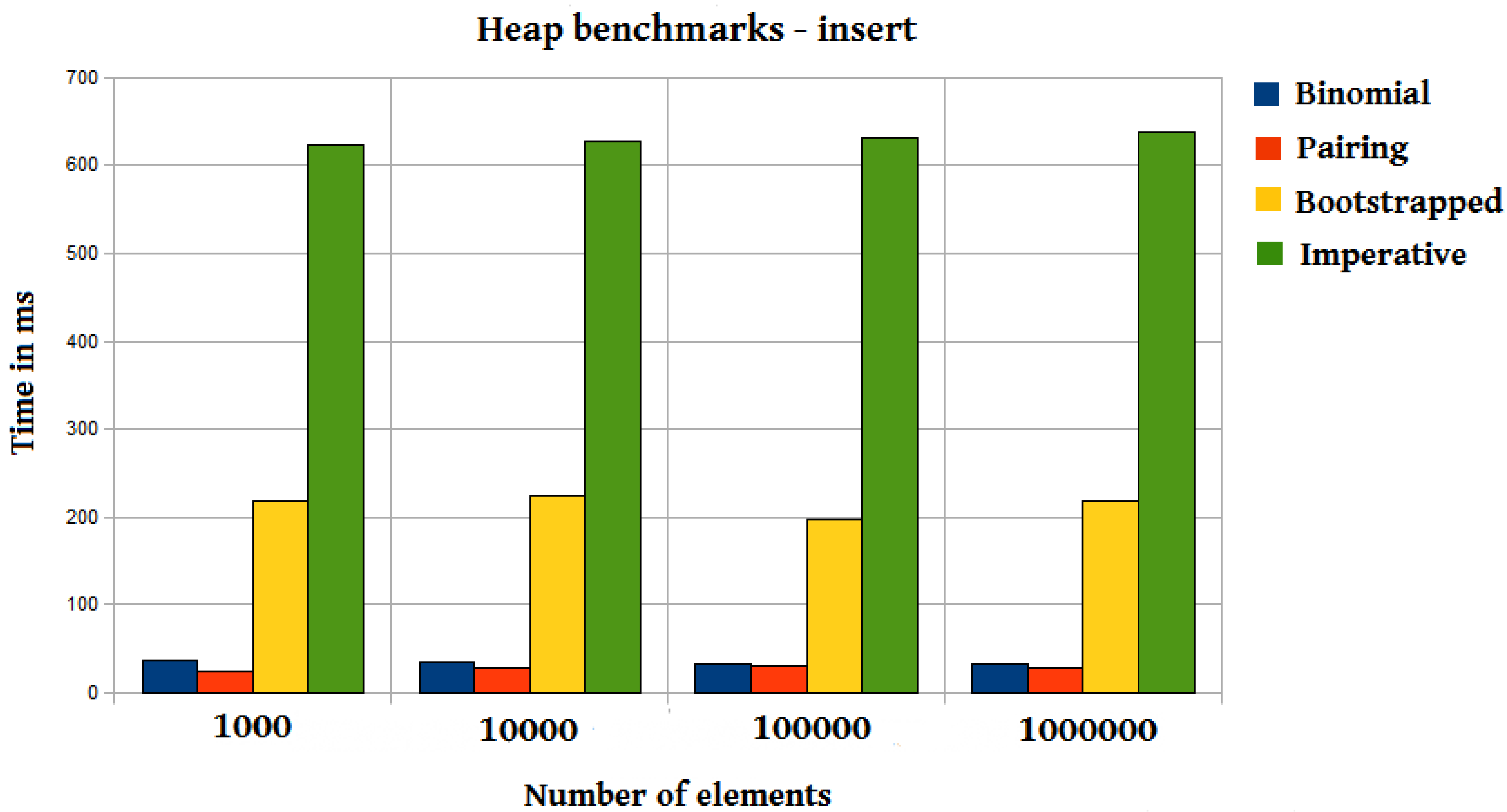

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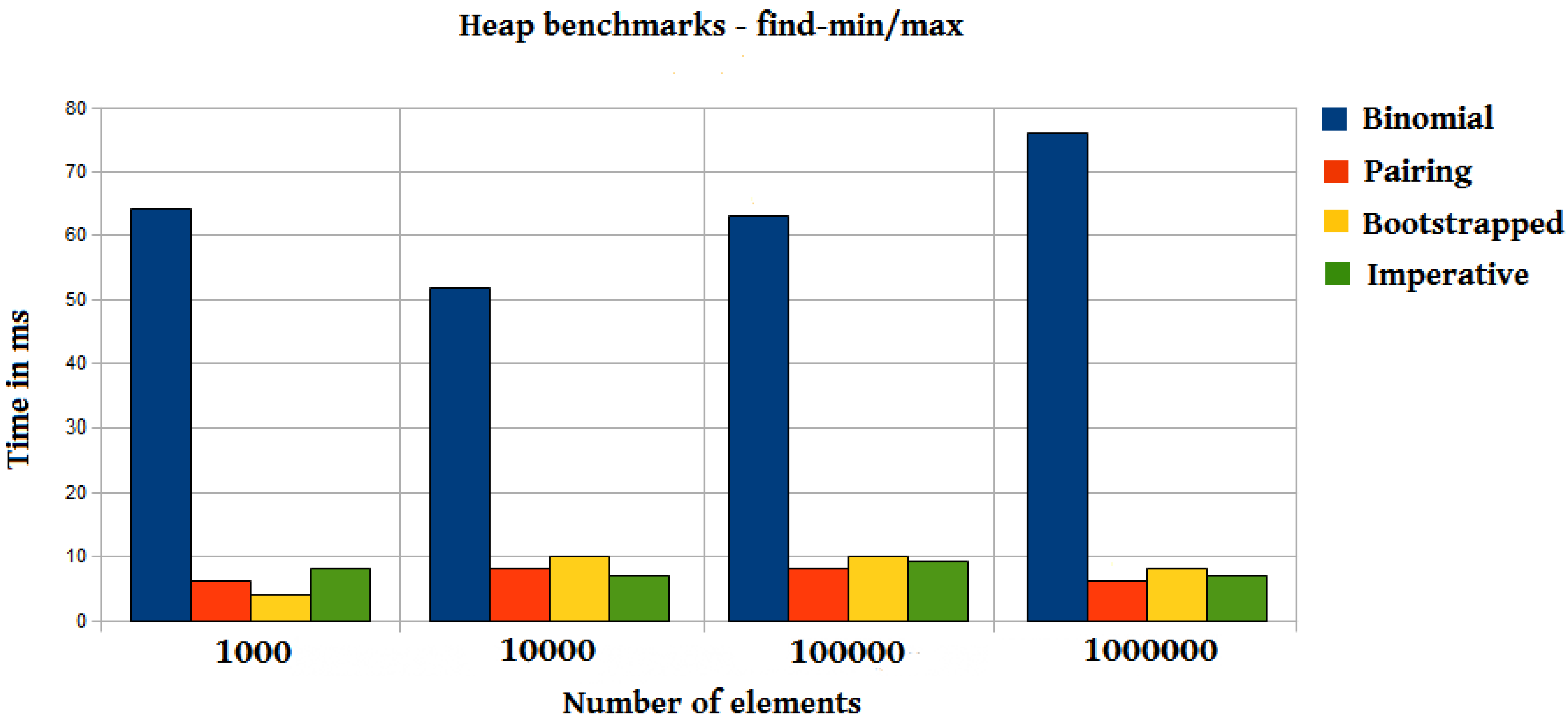

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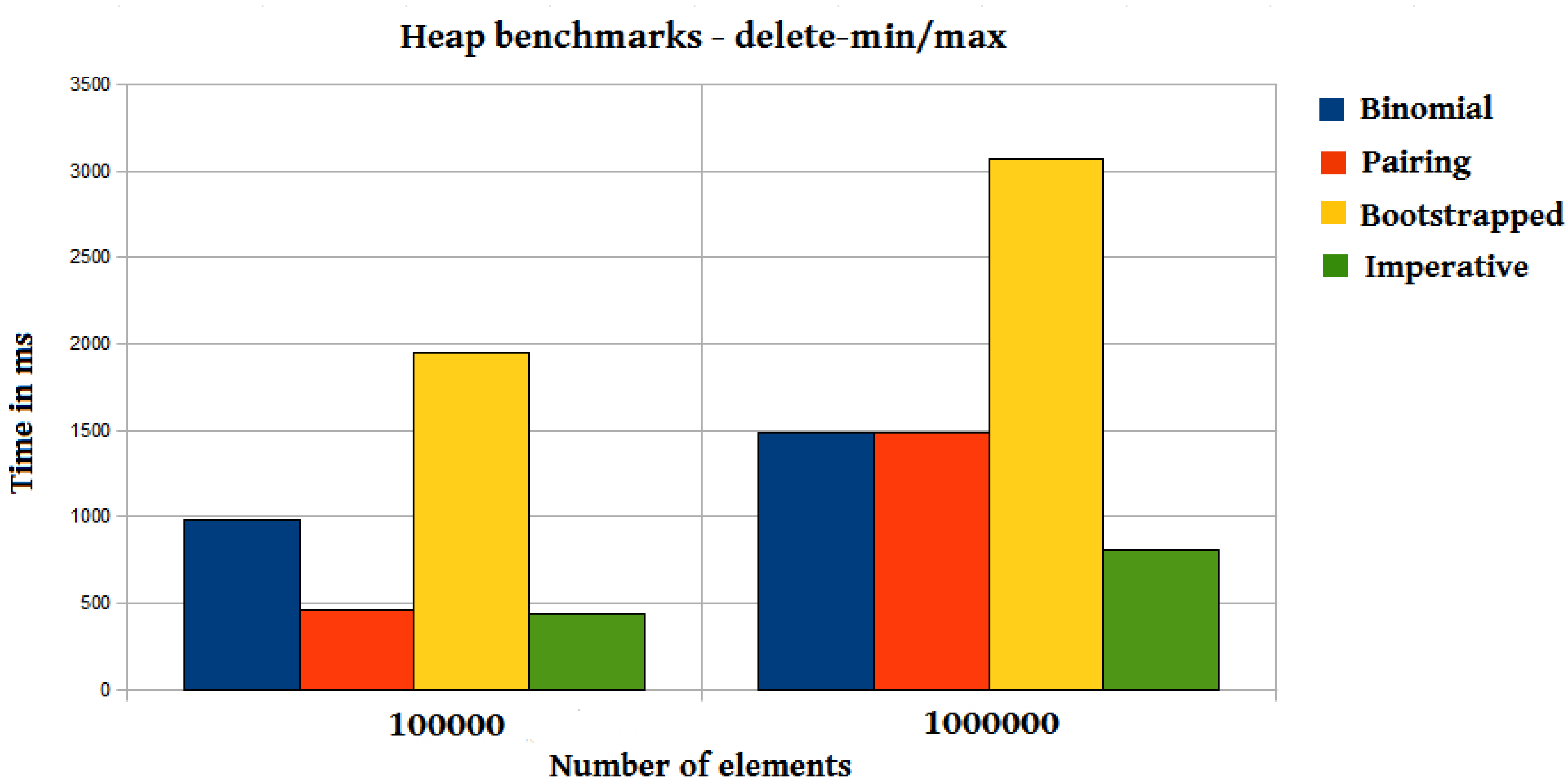

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ML to Typed Racket

ML idioms can be easily ported to Typed Racket

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```
type 'a Queue = int * 'a Stream * int * 'a Stream
```

```
(define-struct: (A) Queue
  ([lenf  : Integer]
   [front : (Stream A)]
   [lenr  : Integer]
   [rear  : (Stream A)]))
```

ML to Typed Racket

ML idioms can be easily ported to Typed Racket

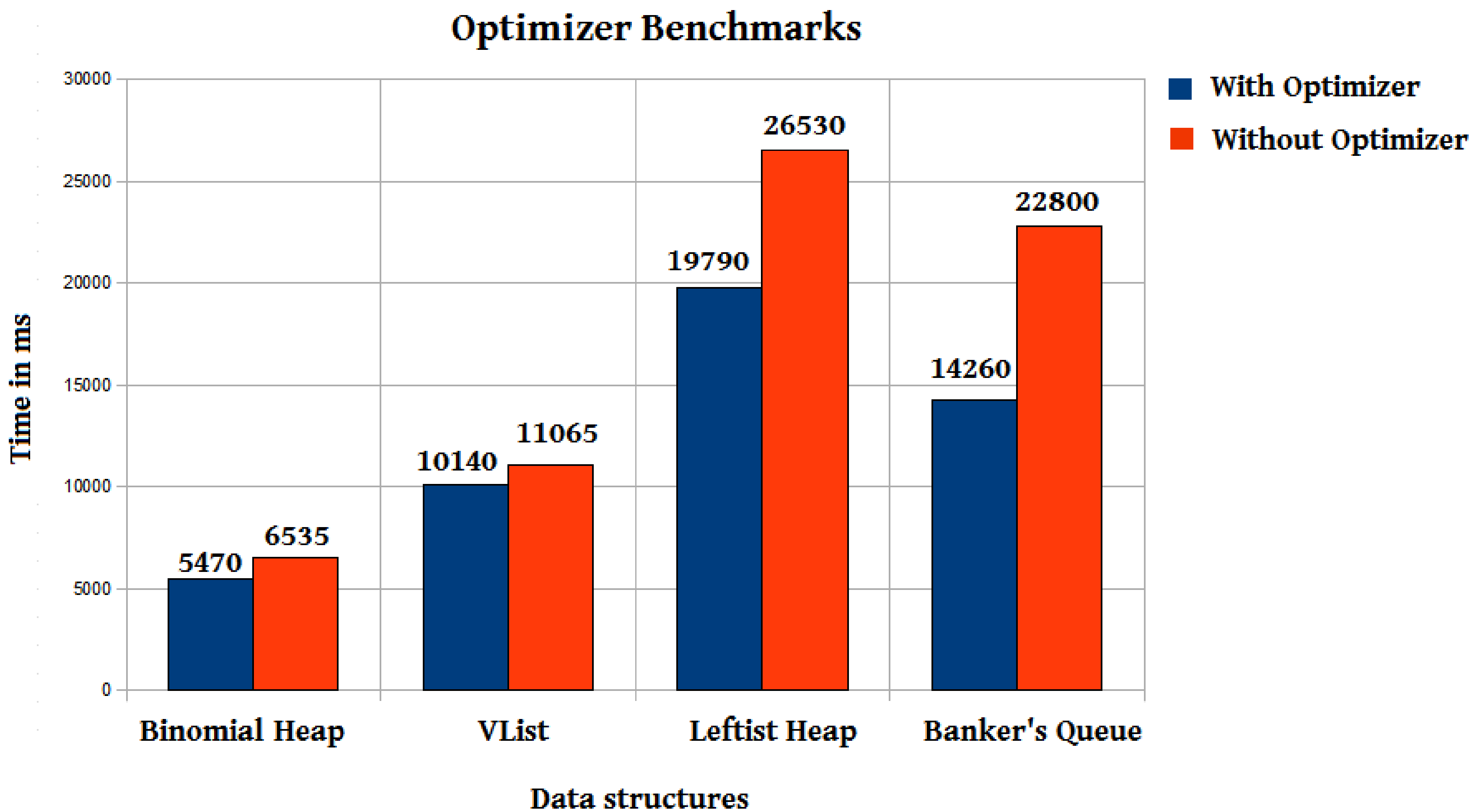
```
type 'a Queue = 'a list * int * 'a list susp * int * 'a list
```

```
(define-struct: (A) Queue
  ([pref  : (Listof A)]
   [lenf  : Integer]
   [front : (Promise (Listof A))]
   [lenr  : Integer]
   [rear  : (Listof A)]))
```

Optimizer in Typed Racket

Optimizer based on type information

Optimizer in Typed Racket



Polymorphic recursion

```
(define-type (Seq A) (Pair A (Seq (Pair A A))))
```

Non-uniform type

Polymorphic recursion

```
(define-type (EP A)  (U A (Pair (EP A) (EP A))))
(define-type (Seq A) (Pair (EP A) (Seq A)))
```

Uniform type

Conclusion

Typed Racket is useful for real-world software.

Functional data structures in Typed Racket are useful and performant.

A comprehensive library of data structures is now available.

Thank you...

Library is available for download from

http://planet.racket-lang.org/