## A Verified Lisp Implementation for A Verified Theorem Prover

Scheme workshop 2016, Nara, Japan

Magnus O. Myreen — University of Cambridge, but now at Chalmers University of Technology Jared Davis — Centaur Technology, Inc., but now at Apple Result:

## A Verified Lisp Implementation for A Verified Theorem Prover

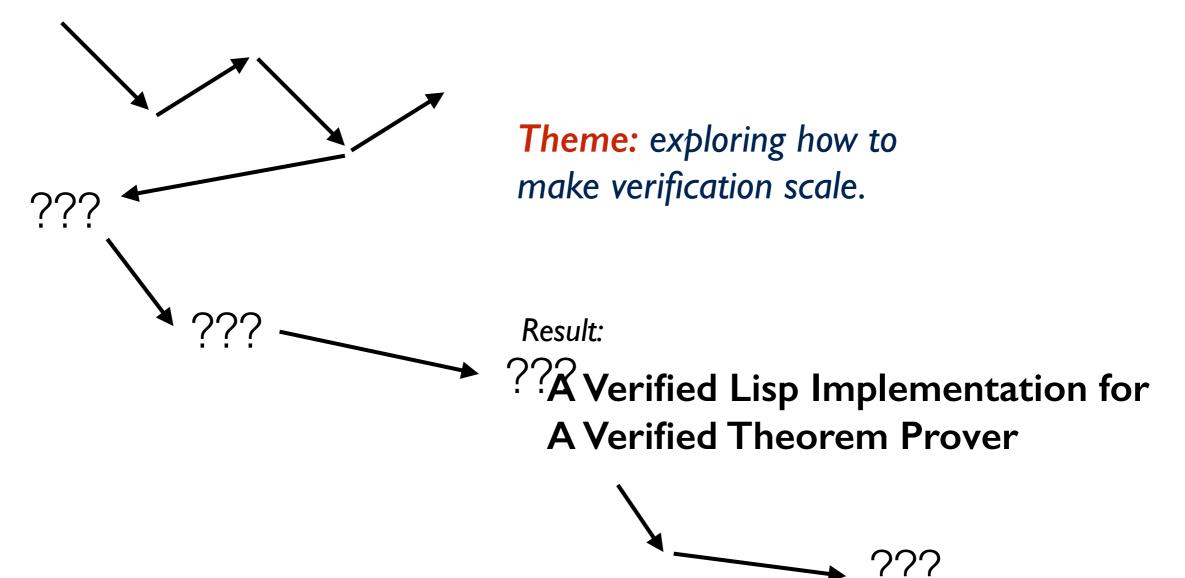
Claim:

The most comprehensive proof-based evidence of a theorem prover's soundness to date.

## This talk: The Journey

2005:

I'm a PhD student working on verification of machine code (factorial, length of a linked list)



## The start:

I'm a PhD student working on verification of machine code (factorial, length of a linked list)

## Context: interactive theorem proving

Aim: to prove deep functional properties of machine code.

Proofs are performed in HOL4 — a fully expansive theorem prover

HOL4 theorem prover



All proofs expand at runtime into primitive inferences in the HOL4 kernel.

The kernel implements the axioms and inference rules of higher-order logic.

## Context: interactive theorem proving



photo idea: Larry Paulsson

## Machine code

Machine code,

#### E1510002 B0422001 C0411002 01AFFFFB

is impossible to read, write or maintain manually.

However, for theorem-prover-based formal verification:

#### machine code is clean and tractable!

Reason:

- ▶ all types are concrete: word32, word8, bool.
- state consists of a few simple components: a few registers, a memory and some status bits.
- each instruction performs only small well-defined updates.

## Challenges of Machine Code

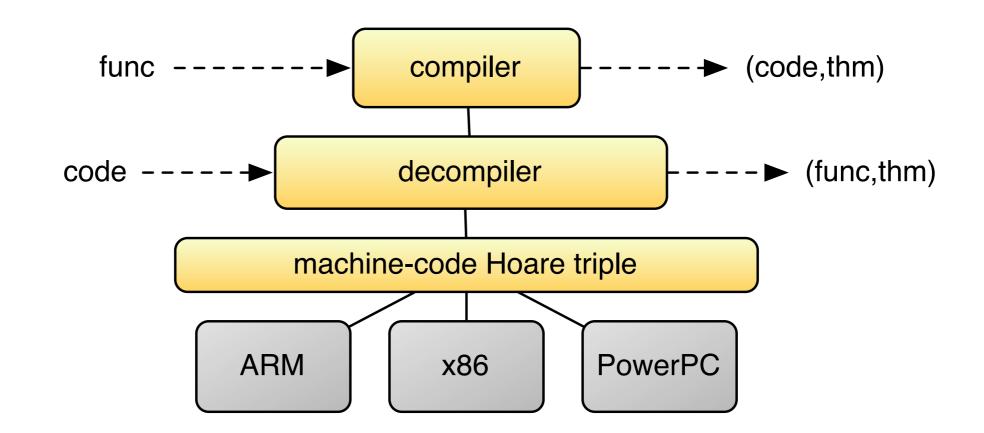


ARM/x86/PowerPC model (1000...10,000 lines each) correctness
{P} code {Q}

- unstructured code
- very low-level and limited resources

## Infrastructure

During my PhD, I developed the following infrastructure:



... each part will be explained in the next slides.

## Hoare triples

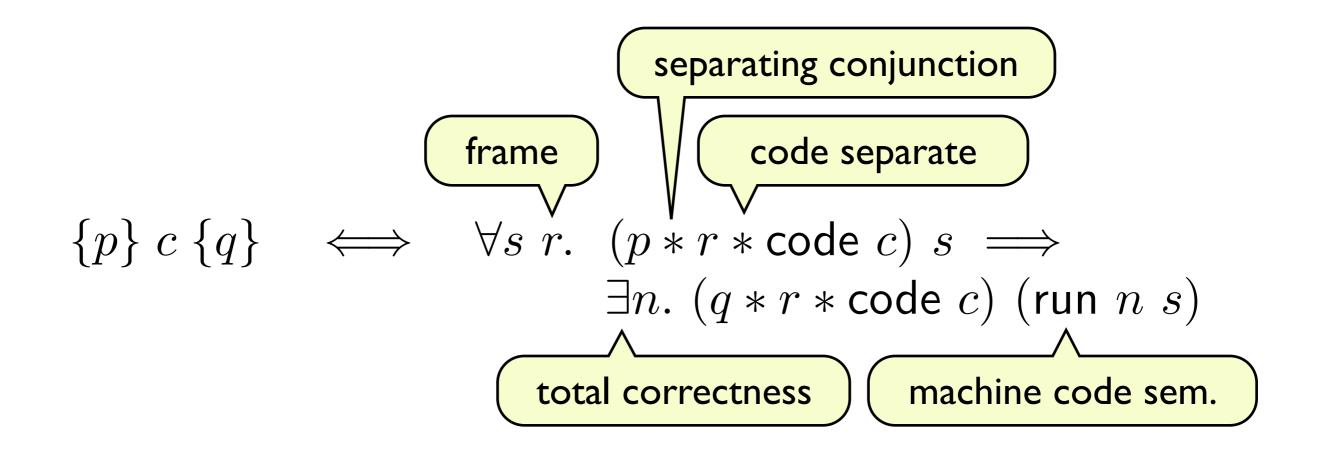
Each model can be evaluated, e.g. ARM instruction add r0,r0,r0 is described by theorem:

|- (ARM\_READ\_MEM ((31 >< 2) (ARM\_READ\_REG 15w state)) state =
 OxE0800000w) ∧ ¬state.undefined ⇒
 (NEXT\_ARM\_MMU cp state =
 ARM\_WRITE\_REG 15w (ARM\_READ\_REG 15w state + 4w)
 (ARM\_WRITE\_REG 0w
 (ARM\_READ\_REG 0w state + ARM\_READ\_REG 0w state) state))</pre>

As a total-correctness machine-code Hoare triple:

- SPEC ARM_MODEL	Informal syntax for this talk:
(aR Ow x * aPC p)	{ R0 x * PC p }
{(p,0xE0800000w)}	<i>p</i> : E0800000
$(aR \ Ow \ (x+x) * aPC \ (p+4w))$	$\{ R0 (x+x) * PC (p+4) \}$

## Definition of Hoare triple



Program logic can be used directly for verification. But direct reasoning in this embedded logic is tiresome.

## Decompiler

Decompiler automates Hoare triple reasoning.

**Example:** Given some ARM machine code,

0:	E3A00000	mov r0, #0
4:	E3510000	L: cmp r1, #0
8:	12800001	addne r0, r0, #1
12:	15911000	ldrne r1, [r1]
16:	1AFFFFFB	bne L

the decompiler automatically extracts a readable function:

$$f(r_0, r_1, m) = \text{let } r_0 = 0 \text{ in } g(r_0, r_1, m)$$
  

$$g(r_0, r_1, m) = \text{if } r_1 = 0 \text{ then } (r_0, r_1, m) \text{ else}$$
  

$$\text{let } r_0 = r_0 + 1 \text{ in}$$
  

$$\text{let } r_1 = m(r_1) \text{ in}$$
  

$$g(r_0, r_1, m)$$

## Decompilation, correct?

Decompiler automatically proves a certificate theorem:

 $f_{pre}(r_0, r_1, m) \Rightarrow \\ \{ (R0, R1, M) \text{ is } (r_0, r_1, m) * PC \ p * S \} \\ p : E3A00000 \ E3510000 \ 12800001 \ 15911000 \ 1AFFFFB \\ \{ (R0, R1, M) \text{ is } f(r_0, r_1, m) * PC \ (p + 20) * S \} \end{cases}$ 

which informally reads:

for any initially value  $(r_0, r_1, m)$  in reg 0, reg 1 and memory, the code terminates with  $f(r_0, r_1, m)$  in reg 0, reg 1 and memory.

## Decompilation verification example

To verify code: prove properties of function f,

 $\forall x \ l \ a \ m. \ list(l, a, m) \Rightarrow f(x, a, m) = (length(l), 0, m)$  $\forall x \ l \ a \ m. \ list(l, a, m) \Rightarrow f_{pre}(x, a, m)$ 

since properties of *f* carry over to machine code via the certificate.

**Proof reuse**: Given similar x86 and PowerPC code:

31C085F67405408B36EBF7

38A000002C140000408200107E80A02E38A500014BFFFFF0

which decompiles into f' and f'', respectively. Manual proofs above can be reused if f = f' = f''.

## Decompilation how to

```
{ R0 i * R1 j * PC p }
p+0:
{ R0 (i+j) * R1 j * PC (p+4) }
```

```
{ R0 i * PC (p+4) }
p+4 :
{ R0 (i >> I) * PC (p+8) }
```

```
{ LR lr * PC (p+8) }
p+8 :
{ LR lr * PC lr }
```

```
How to decompile:
```

<b>e0810000</b> 0	add	r0,	r1,	r0
e1a3300000	lsr	r0,	r0,	#1
e12fffffiee	bx	lr		

- I. derive Hoare triple theorems using Cambridge ARM model
- 2. compose Hoare triples
- 3. extract function
- (Loops result in recursive functions.)

```
{ R0 i * RI j * LR lr * PC p }
p : e0810000 e1a000a0 e12fff1e
{ R0 ((i+j)>>I) * RI j * LR lr * PC lr }
```

3 → avg (i,j) = (i+j)>>1

## Decompiler cont.

Implementation:

- ML program which fully automatically performs forward proof
- no heuristics and no dangling proof obligations
- Ioops result in tail-recursive functions

Case studies:

- verified copying garbage collector
- bignum library routines

## Part 2:

I want more automation and abstraction!

## Proof-producing compilation

Synthesis often more practical. Given function f,

 $f(r_1) = \text{if } r_1 < 10 \text{ then } r_1 \text{ else let } r_1 = r_1 - 10 \text{ in } f(r_1)$ 

our *compiler* generates ARM machine code:

E351000A	L:	cmp r1,#10
2241100A		<pre>subcs r1,r1,#10</pre>
2AFFFFFC		bcs L

and automatically proves a certificate HOL theorem:

 $\vdash \{ R1 r_1 * PC p * s \}$ p: E351000A 2241100A 2AFFFFC  $\{ R1 f(r_1) * PC (p+12) * s \}$ 

## Compilation, example cont.

One can prove properties of f since it lives inside HOL:

 $\vdash \forall x. \ f(x) = x \bmod 10$ 

Properties proved of f translate to properties of the machine code:

 $\vdash \{ \text{R1} \ r_1 * \text{PC} \ p * \text{s} \}$  p : E351000A 2241100A 2AFFFFC $\{ \text{R1} \ (r_1 \mod 10) * \text{PC} \ (p+12) * \text{s} \}$ 

Additional feature: the compiler can use the above theorem to extend its input language with: let  $r_1 = r_1 \mod 10$  in \_

### Implementation

To compile function f:

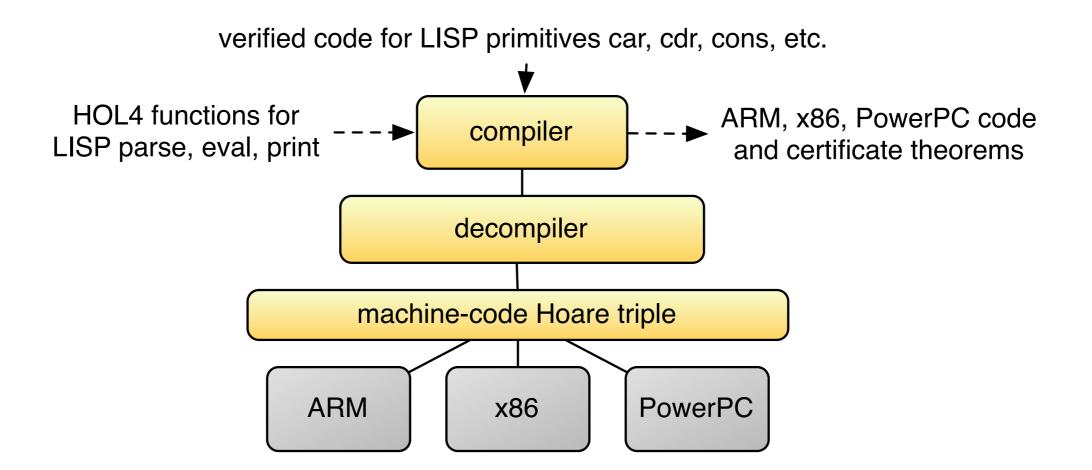
- 1. generate, without proof, code from input f;
- 2. decompile, with proof, a function f' from generated code;
- 3. prove f = f'.

Features:

- code generation completely separate from proof
- supports many light-weight optimisations without any additional proof burden: instruction reordering, conditional execution, dead-code elimination, duplicate-tail elimination, ...
- allows for significant user-defined extensions

## Infrastructure again

Idea: create LISP implementations via compilation.



## Lisp formalised

LISP s-expressions defined as data-type SExp:

Num :  $\mathbb{N} \rightarrow SExp$ Sym : string  $\rightarrow SExp$ Dot : SExp  $\rightarrow SExp \rightarrow SExp$ 

LISP primitives were defined, e.g.

$$cons x y = Dot x y$$
$$car (Dot x y) = x$$
$$plus (Num m) (Num n) = Num (m + n)$$

The semantics of LISP evaluation was taken to be Gordon's formalisation of 'LISP 1.5'-like evaluation

## Extending the compiler

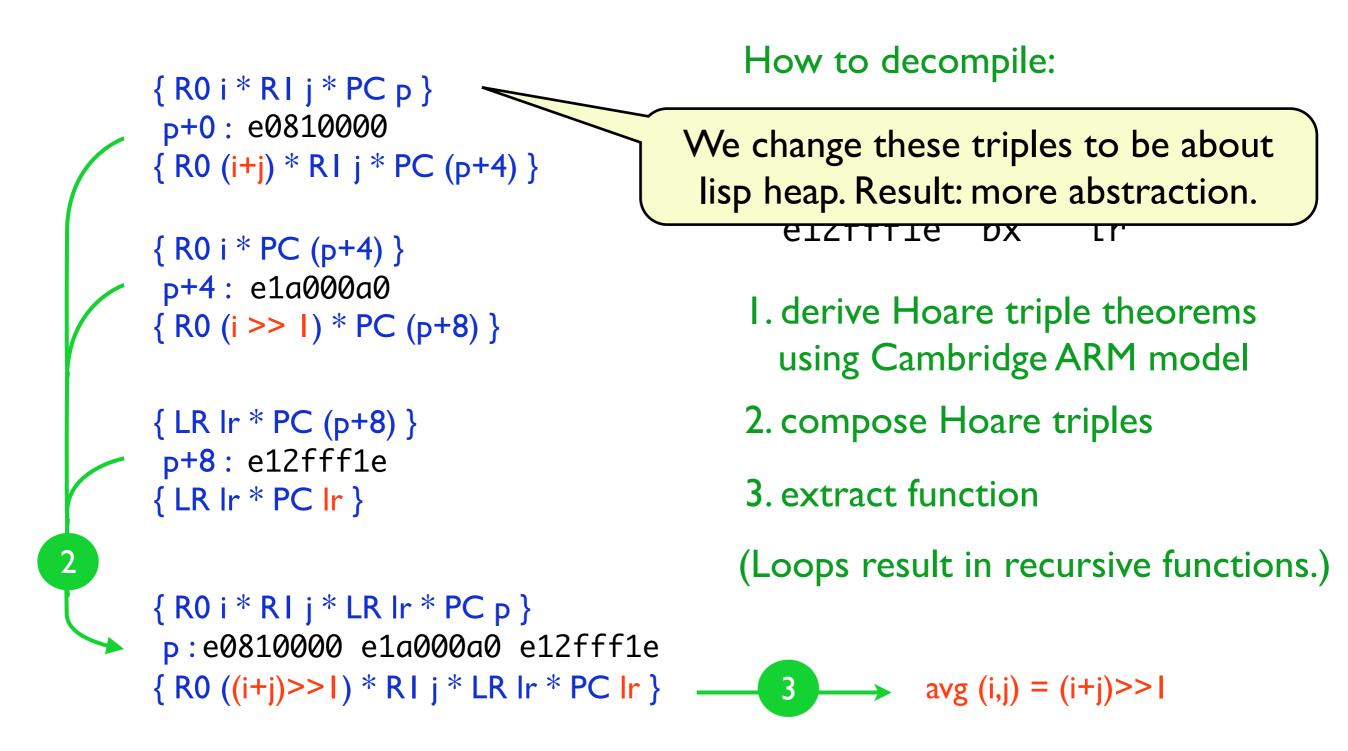
We define heap assertion 'lisp  $(v_1, v_2, v_3, v_4, v_5, v_6, I)$ ' and prove implementations for primitive operations, e.g.

is\_pair  $v_1 \Rightarrow$ { lisp  $(v_1, v_2, v_3, v_4, v_5, v_6, l) * pc p$  } p : E5934000{ lisp  $(v_1, car v_1, v_3, v_4, v_5, v_6, l) * pc (p + 4)$  } size  $v_1 + size v_2 + size v_3 + size v_4 + size v_5 + size v_6 < l \Rightarrow$ { lisp  $(v_1, v_2, v_3, v_4, v_5, v_6, l) * pc p$  } p : E50A3018 E50A4014 E50A5010 E50A600C ...{ lisp  $(cons v_1 v_2, v_2, v_3, v_4, v_5, v_6, l) * pc (p + 332)$  }

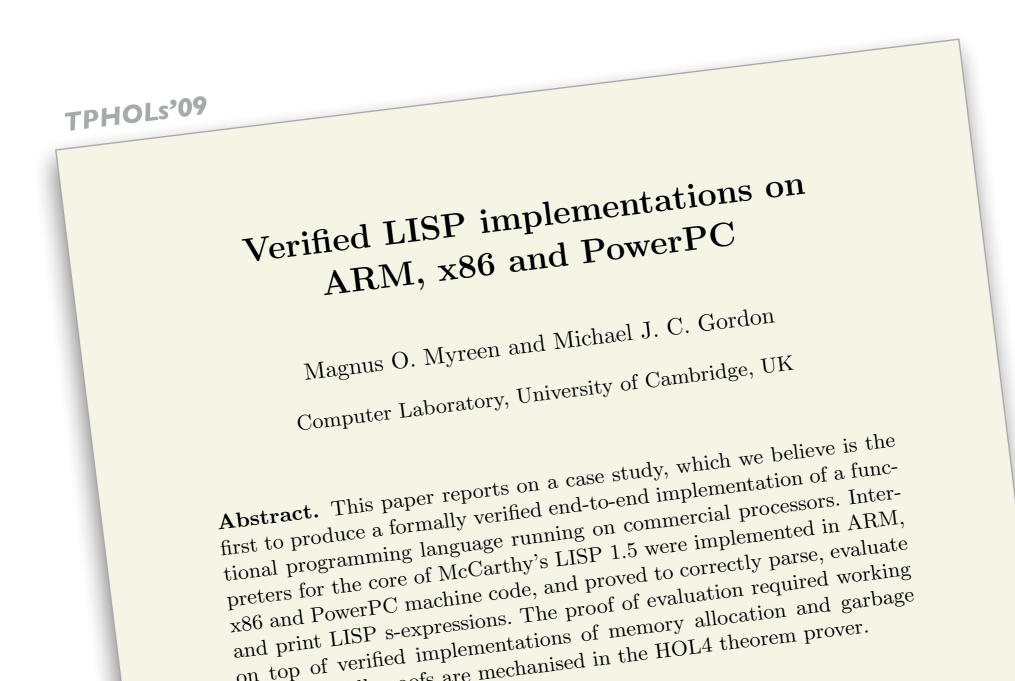
with these the compiler understands:

let  $v_2 = \operatorname{car} v_1$  in ... let  $v_1 = \operatorname{cons} v_1 v_2$  in ...

## Reminder



## The final case study of my PhD



## Running the Lisp interpreter



```
Nintendo DS lite (ARM) MacBook (x86) old MacMini (PowerPC)
```

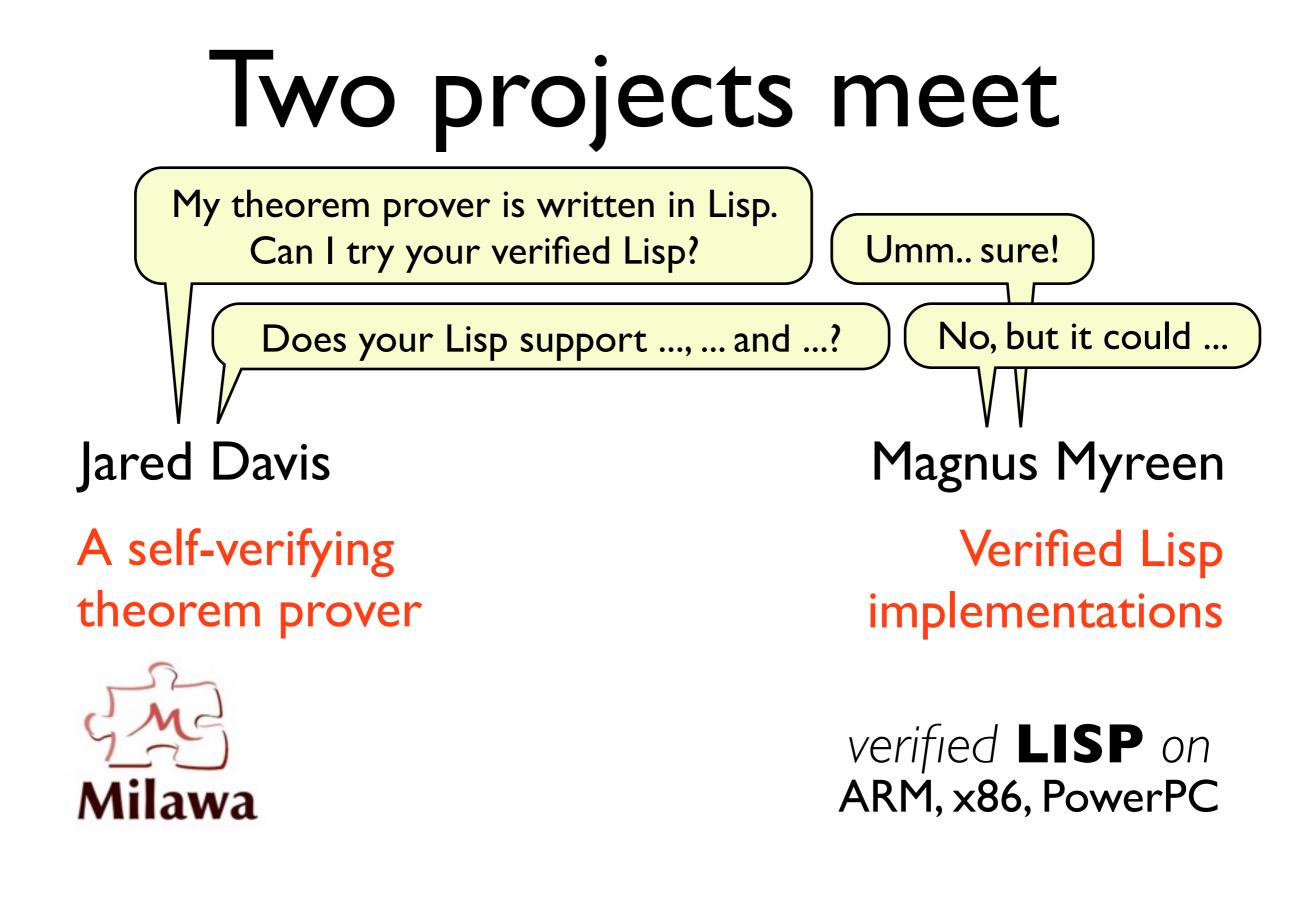
```
(pascal-triangle '((1)) '6)
```

returns:

```
((1 6 15 20 15 6 1)
(1 5 10 10 5 1)
(1 4 6 4 1)
(1 3 3 1)
(1 2 1)
(1 1)
(1))
```

## Part 3:

A sudden need for a serious Lisp implementation.



# Running Milawa



Milawa's bootstrap proof:

hopelessly "toy"

- 4 gigabyte proof file:
   >500 million unique conses
- takes 16 hours to run on a state-of-the-art runtime (CCL)

verified **LISP** on **ARM, x86, PowerPC** 

# Running Milawa



Milawa's bootstrap proof:

- 4 gigabyte proof file:
   >500 million unique conses
- takes 16 hours to run on a state-of-the-art runtime (CCL)

Jitawa: verified LISP with JIT compiler

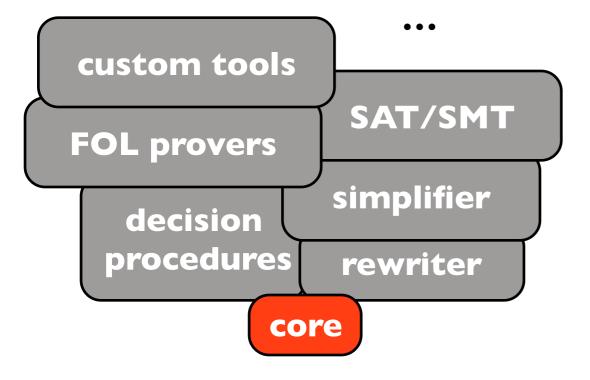
#### **Result:**

a new verified Lisp which is able to host the Milawa thm prover

# A short introdution to

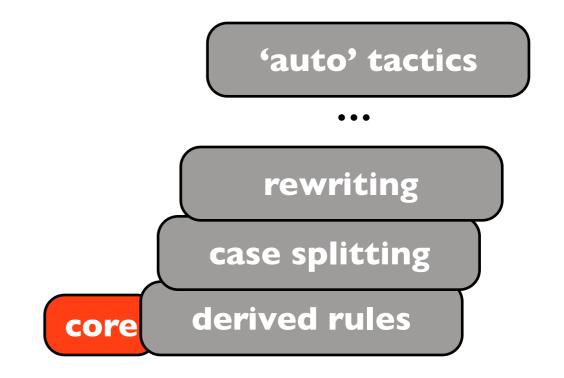
- Milawa is styled after theorem provers such as NQTHM and ACL2,
- has a small trusted logical kernel similar to LCF-style provers,
- ... but does not suffer the performance hit of LCF's fully expansive approach.

## Comparison with LCF approach



#### LCF-style approach

- all proofs pass through the core's primitive inferences
- extensions steer the core



#### the Milawa approach

- all proofs must pass the core
- the core proof checker can be replaced at runtime

# Requirements on runtime

Milawa uses a subset of Common Lisp which

is for most part first-order pure functions over natural numbers, symbols and conses,

uses primitives: cons car cdr consp natp symbolp
 equal + - < symbol-< if</pre>

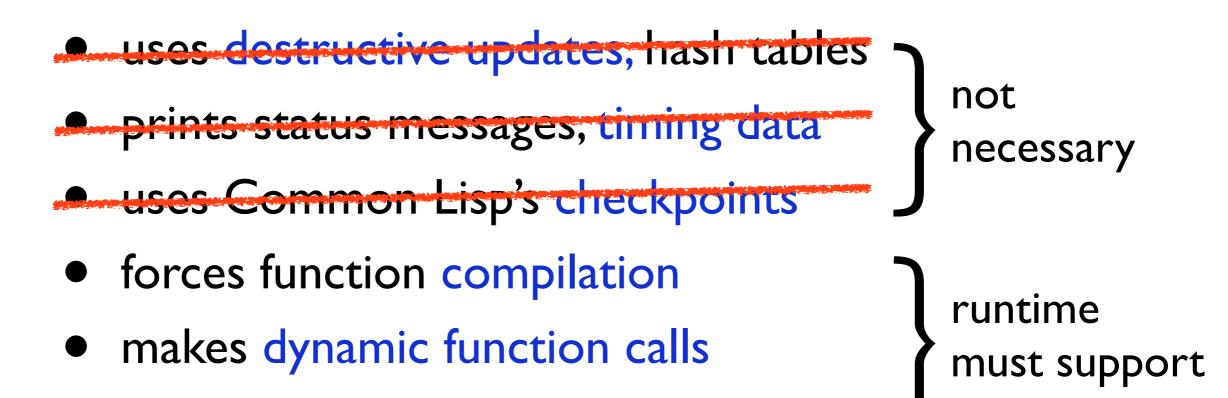
macros: or and list let let\* cond first second third fourth fifth

and a simple form of lambda-applications.

(Lisp subset defined on later slide.)

# Requirements on runtime

...but Milawa also



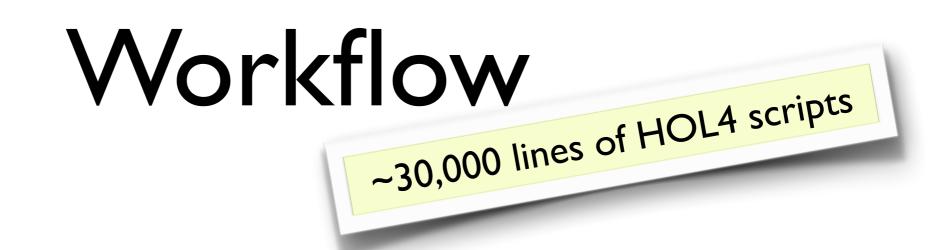
• can produce runtime errors

(Lisp subset defined on later slide.)

# Runtime must scale

#### Designed to scale:

- just-in-time compilation for speed
  - functions compile to native code
- target 64-bit x86 for heap capacity
  - space for 2<sup>31</sup> (2 billion) cons cells (16 GB)
- efficient scannerless parsing + abbreviations
  - must cope with 4 gigabyte input
- graceful exits in all circumstances
  - allowed to run out of space, but must report it



- I. specified input language: syntax & semantics
- 2. verified necessary algorithms, e.g.
  - compilation from source to bytecode
  - parsing and printing of s-expressions
  - copying garbage collection
- 3. proved refinements from algorithms to x86 code
- 4. plugged together to form read-eval-print loop

## AST of input language

term	::=     	Const sexp set Var string App func (term list) If term term term	exp	::=   	Val <i>num</i> Sym <i>string</i> Dot <i>sexp sexp</i>
		LambdaApp ( <i>string</i> list) <i>term</i> ( <i>term</i> list Or ( <i>term</i> list)	st)		
		And $(term list)$		(macro)	)
		List $(term \ list)$		(macro)	)
		Let $((string \times term) \text{ list}) term$		(macro)	)
		LetStar $((string \times term) \text{ list}) term$		(macro)	)
		Cond $((term \times term) \text{ list})$		(macro)	)
		First <i>term</i>   Second <i>term</i>   Third <i>term</i>		(macro)	)
		Fourth term   Fifth term		(macro)	)
func	=:: 	Define   Print   Error   Funcall PrimitiveFun <i>primitive</i>   Fun <i>string</i>			
primitive	::=   	Equal   Symbolp   SymbolLess Consp   Cons   Car   Cdr   Natp   Add   Sub   Less			

## compile: $AST \rightarrow bytecode list$

bytecode

Pop PopN num PushVal num PushSym *string* LookupConst *num* Load *num* Store *num* DataOp *primitive* Jump num JumpIfNil *num* DynamicJump Call *num* DynamicCall Return Fail Print Compile

pop one stack element pop n stack elements push a constant number push a constant symbol push the nth constant from system state push the nth stack element overwrite the nth stack element add, subtract, car, cons, ... jump to program point nconditionally jump to njump to location given by stack top static function call (faster) dynamic function call (slower) return to calling function signal a runtime error print an object to stdout compile a function definition

#### How do we get just-in-time compilation?

Treating code as data:

 $\forall p \ c \ q. \quad \{p\} \ c \ \{q\} \ = \ \{p \ast \mathsf{code} \ c\} \ \emptyset \ \{q \ast \mathsf{code} \ c\}$ 

#### Definition of Hoare triple:

$$\begin{array}{rcl} \{p\} \ c \ \{q\} & = & \forall s \ r. & (p \ast r \ast \mathsf{code} \ c) \ s \implies \\ & \exists n. \ (q \ast r \ast \mathsf{code} \ c) \ (\mathsf{run} \ n \ s) \end{array}$$

# I/O and efficient parsing

Jitawa implements a read-eval-print loop:

Use of external C routines adds assumptions to proof:

- reading next string from stdin
- printing null-terminated string to stdout

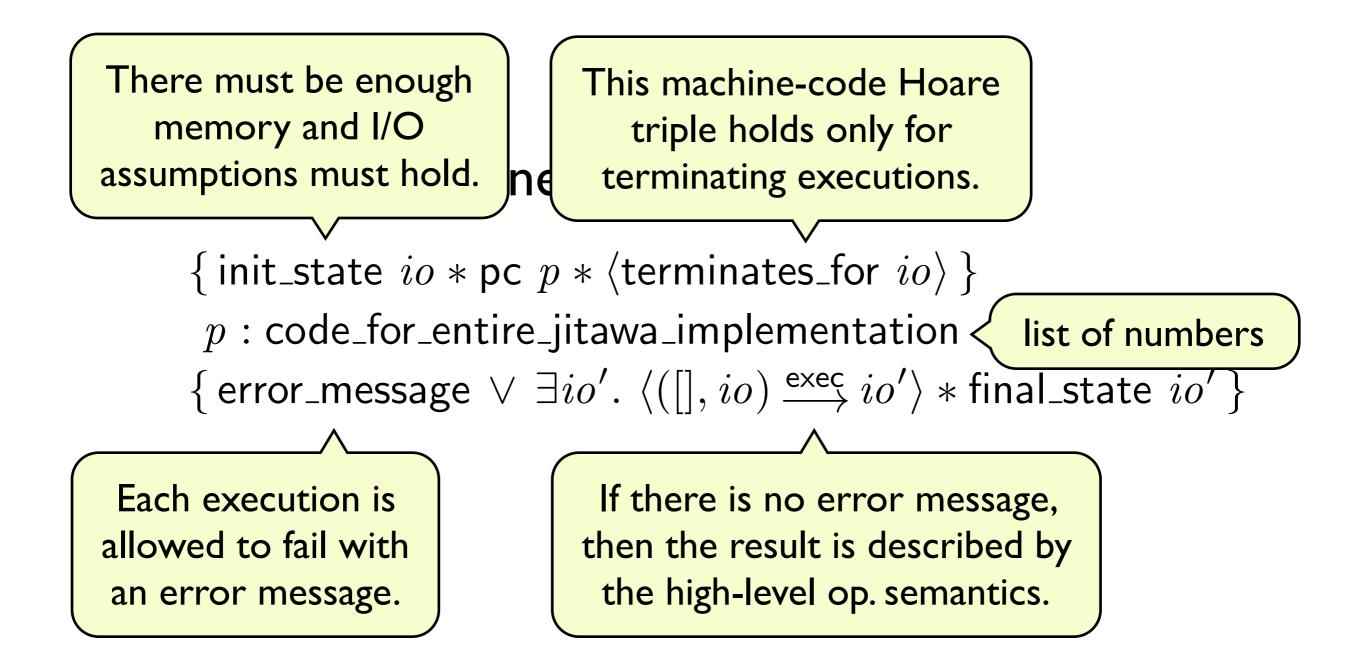
# Read-eval-print loop

- Result of reading lazily, writing eagerly
- Eval = compile then jump-to-compiled-code
- Specification: read-eval-print until end of input

$$\neg \text{is\_empty (get\_input } io) \land \\ \text{next\_sexp (get\_input } io)) = (s, rest) \land \\ (\text{sexp2term } s, [], k, \text{set\_input } rest \; io) \xrightarrow{\text{ev}} (ans, k', io') \land \\ (k', \text{append\_to\_output (sexp2string } ans) \; io') \xrightarrow{\text{exec}} io'' \\ (k, io) \xrightarrow{\text{exec}} io''$$

is\_empty (get\_input *io*)  $(k, io) \xrightarrow{\text{exec}} io$ 

## Correctness theorem



## Verified code

#### \$ cat verified\_code.s

/\* Machine code automatically extracted from a HOL4 theorem. \*/

\*/

/\* The code consists of 7423 instructions (31840 bytes).

.byte	0x48,	0x8B,	0x5F,	0x18		
.byte	0x4C,	0x8B,	0x7F,	0x10		
.byte	0x48,	0x8B,	0x47,	0x20		
.byte	0x48,	0x8B,	0x4F,	0x28		
.byte	0x48,	Øx8B,	0x57,	0x08		
.byte	0x48,	0x8B,	0x37			
.byte	0x4C,	0x8B,	0x47,	0x60		
.byte	0x4C,	Øx8B,	0x4F,	0x68		
.byte	0x4C,	Øx8B,	0x57,	0x58		
.byte	0x48,	0x01,	ØxC1			
.byte	0xC7,	0x00,	0x04,	0x4E,	0x49,	0x4C
.byte	0x48,	0x83,	0xC0,	0x04		
.byte	0xC7,	0x00,	0x02,	0x54,	0x06,	0x51
.byte	0x48,	0x83,	0xC0,	0x04		

# Running Milawa on Jitawa

Running Milawa's 4-gigabyte booststrap process:

CCLI6 hoursJitawa's compiler performsSBCL22 hoursalmost no optimisations.Jitawa128 hours(8x slower than CCL)

Parsing the 4 gigabyte input:

CCL716 seconds(9x slower than Jitawa)Jitawa79 seconds

### Part 4:

The end-to-end result

# Proving Milawa sound

semantics of Milawa's logic

inference rules of Milawa's logic

Milawa theorem prover (kernel approx. 2000 lines of Milawa Lisp)

Milawa

Jitawa

verified

LISP

Lisp semantics

Lisp implementation (x86) (approx. 7000 64-bit x86 instructions)

semantics of x86-64 machine

proving soundness of the source code

verification of a Lisp implementation

Assumes x86 model, C wrapper, OS, hardware



Milawa theorem prover (kernel approx. 2000 lines of Milawa Lisp)

https://raw.githubusercontent.com/HOL-Theorem-Prover/HOL/master/examples/theorem-prover/milawa-prover/core.lisp

## Proving the top-level theorem

#### The top-level theorem:

relates the logic's semantics with the execution of the x86 machine code.

#### Steps:

- A. formalise Milawa's logic
  - syntax, semantics, inference, soundness
- **B**. prove that Milawa's kernel is faithful to the logic
  - run the Lisp parser (in the logic) on Milawa's kernel
  - translate (with proof) deep embedding into shallow
  - prove that Milawa's (reflective) kernel is faithful to logic
- C. connect the verified Lisp implementation
  - compose with the correctness thm for Lisp system

## Theorem: Milawa is sound down to x86

There must be enough memory and input is Milawa's kernel followed by call to main for some *input*.

 $\forall input \ pc.$ 

{ init\_state (milawa\_implementation ++ "(milawa-main 'input)") \* pc pc } pc : code\_for\_entire\_jitawa\_implementation

{ error\_message  $\lor$  (let  $result = compute_output$  (parse input) in  $\land$   $\land$  every\_line line\_ok  $result \land *$ 

final\_state (output\_string result ++ "SUCCESS")) }

Machine code terminates either with error message, or ...

... output lines that are all true w.r.t. the semantics of the logic.

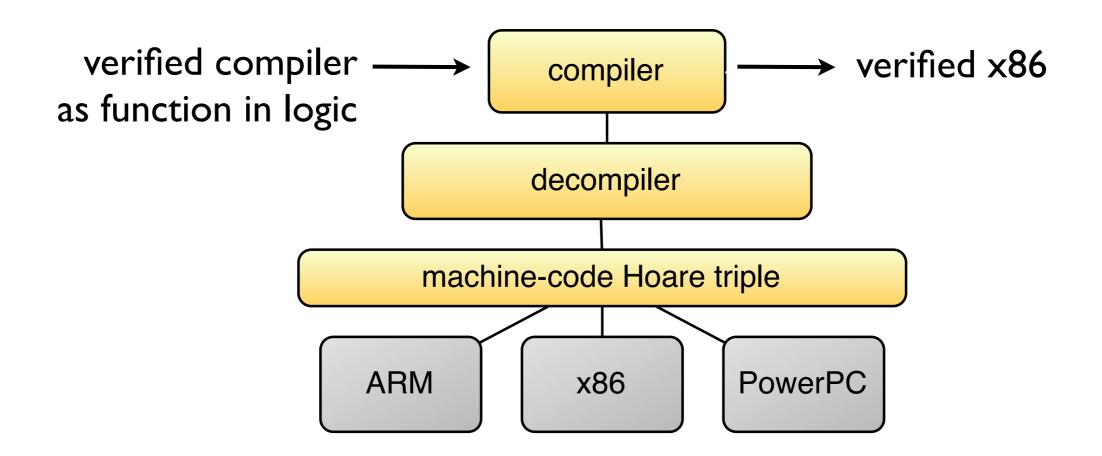
line\_ok 
$$(\pi, l) = (l = "NIL") \lor (\exists n. (l = "(PRINT (n ...))") \land is_number n) \lor (\exists \phi. (l = "(PRINT (THEOREM \phi))") \land context_ok \pi \land \models_{\pi} \phi)$$

### **Final Part:**

Learning from the mistakes. Doing it better.

## A better compiler compiler?

The x86 for the compile function was produced as follows:



A bit cumbersome....

...should have compiled the verified compiler using itself!

### Bootstrapping the compiler

Instead: we should bootstrap the verified compile function, i.e. evaluate the compiler on a deep embedding of itself within the logic:

#### EVAL ``compile COMPILE``

derives a theorem:

compile COMPILE = compiler-as-machine-code



Ramana Kumar (Uni. Cambridge)



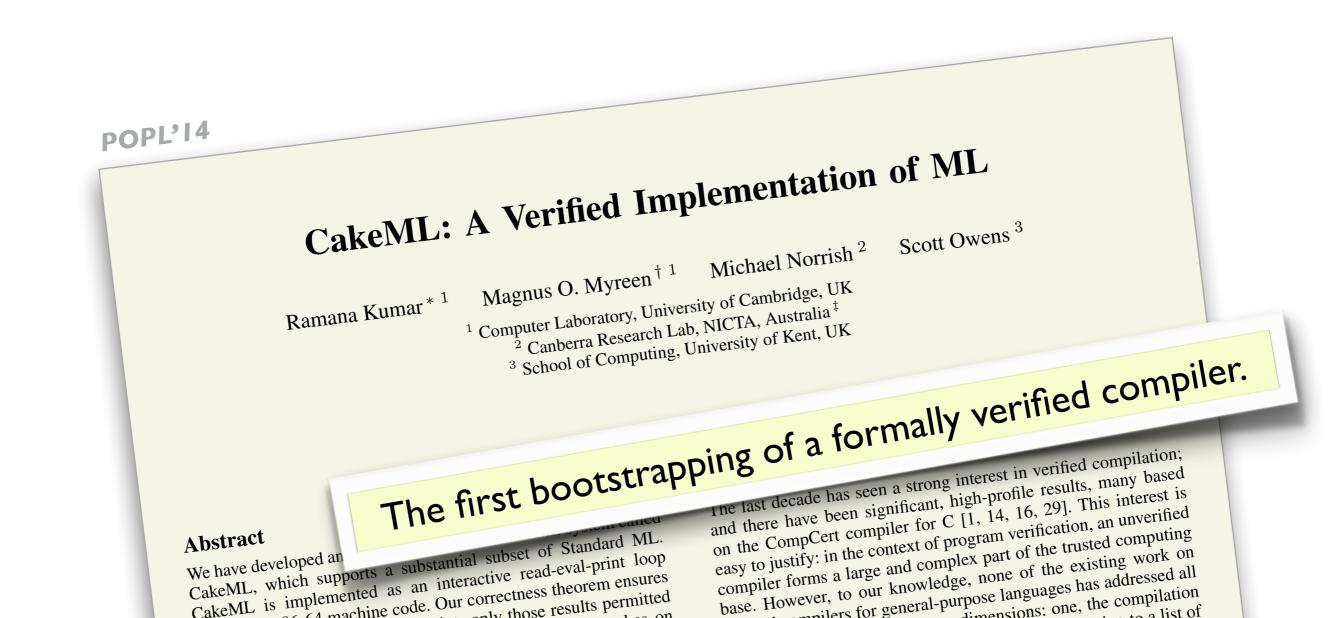
Magnus Myreen (Uni. Cambridge)



Michael Norrish (NICTA, ANU)



Scott Owens (Uni. Kent)



### **Tomorrow at ICFP!**





Magnus Myreen



Yong Kiam Tan

Ramana Kumar

Anthony Fox



Scott Owens



Michael Norrish

## Looking back...

2005:

I'm a PhD student working on verification of machine code (factorial, length of a linked list)

??? **Result: A Verified Lisp Implementation for A Verified Theorem Prover** 

2005:

I'm a PhD student working on verification of machine code (factorial, length of a linked list)

basic reasoning about real machine code powerful automation verification of garbage collectors synthesis from (abstract) functional specs verified Lisp interpreters verified just-in-time compiler for Lisp Result:

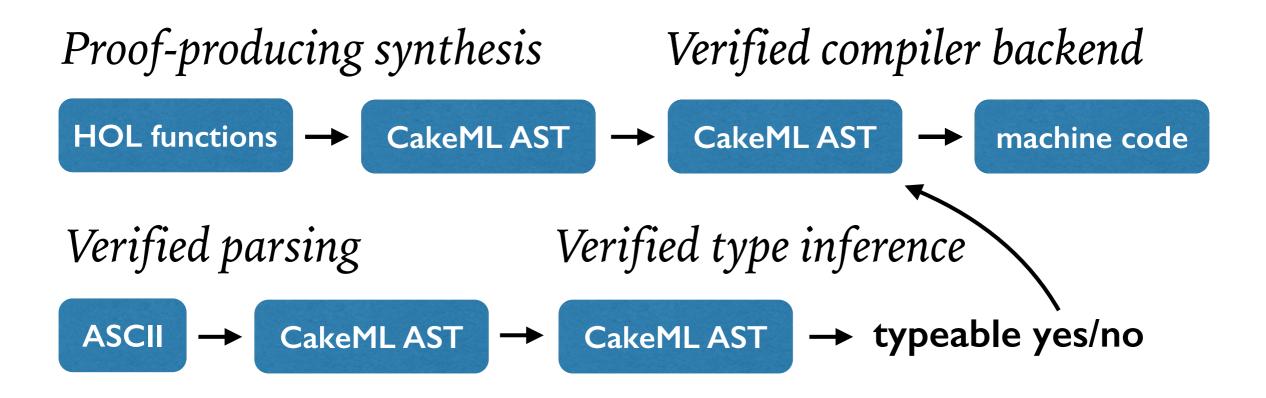
> A Verified Lisp Implementation for A Verified Theorem Prover



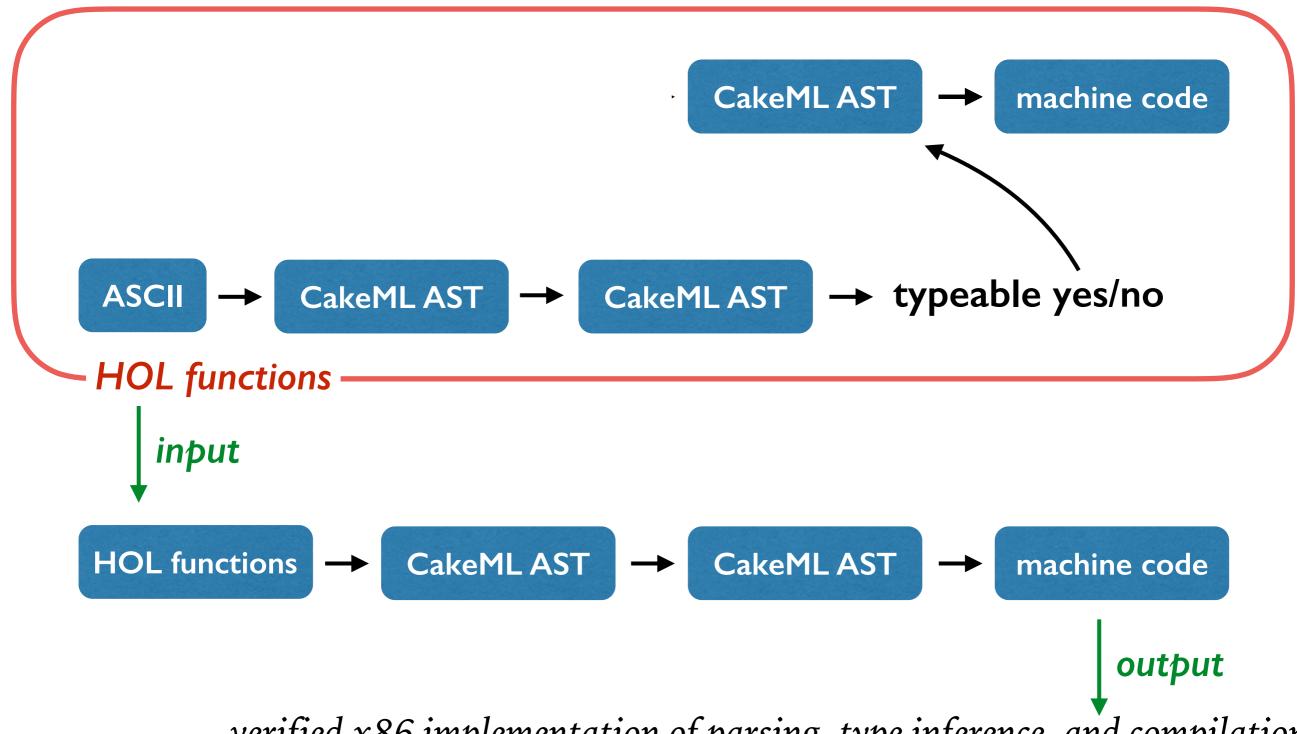
Thank you for inviting me!

verified compiler bootstrapping (ML)

## Intuition for Bootstrapping



## Intuition for Bootstrapping



verified x86 implementation of parsing, type inference, and compilation